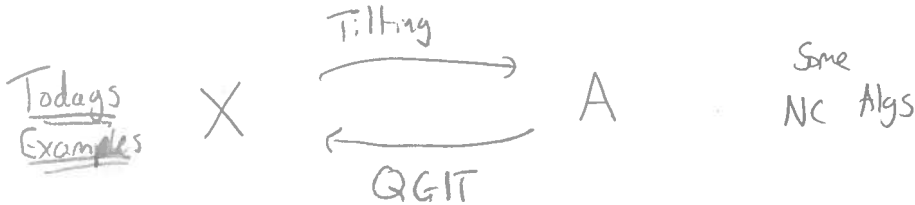
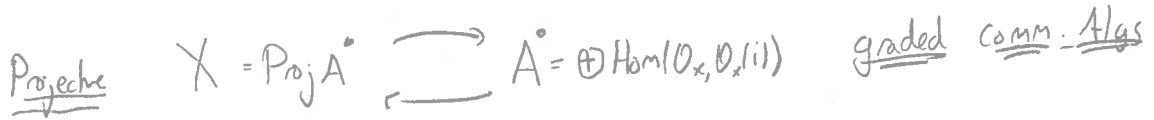
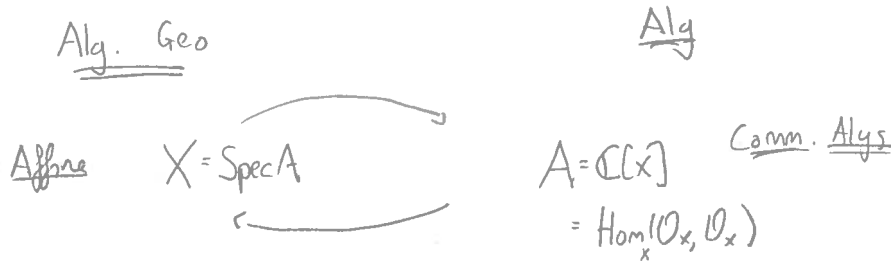


Geo club

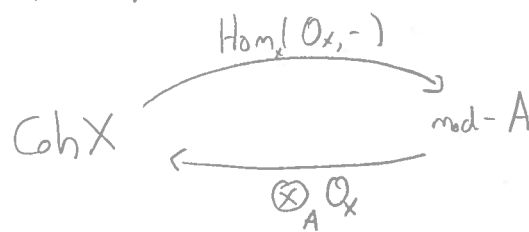
Tilting + Quiver GIT

1 // Intro



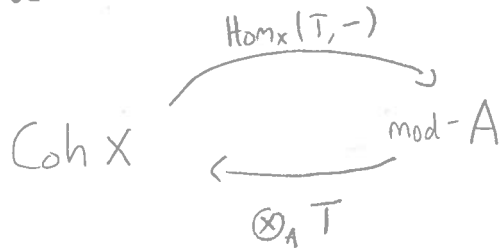
2 // Tilting

Affine Case (Serre)



is an equiv of abelian cats

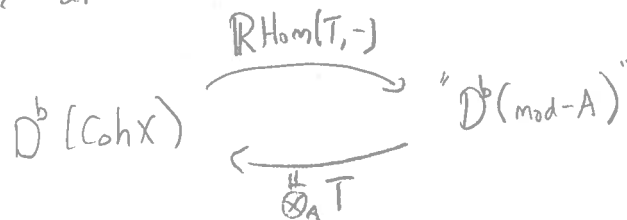
Generalise ... Choose T a v.b. on X



$A = \text{Hom}_x(T, T)$

is basically never an equiv of cats unless X is affine

Instead look at



$A = \text{RHom}(T, T)$

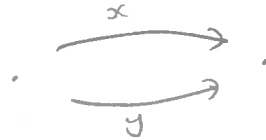
Defⁿ T is a tilting bundle if

1) $\text{RHom}(T, T) = \text{Hom}(T, T) = A$

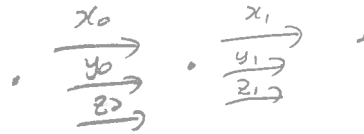
2) $\text{RHom}(T, -)$ is an equiv. of derived cats

Examples

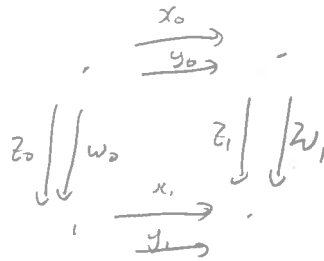
$\mathbb{P}^1 \quad \mathcal{O} \oplus \mathcal{O}(1)$



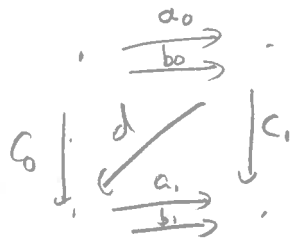
$\mathbb{P}^2 \quad \mathcal{O} \oplus \mathcal{O}(1) \oplus \mathcal{O}(2)$



$\mathbb{P}^1 \times \mathbb{P}^1 \quad \mathcal{O} \oplus \mathcal{O}(0,1) \oplus \mathcal{O}(1,0) \oplus \mathcal{O}(1,1)$



$\text{Bl}_{pt} \mathbb{P}^1 \cong \sum_{\mathbb{C}} \mathbb{P}^1 \times \mathbb{P}^2$

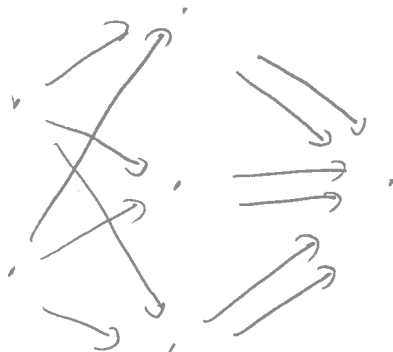


$\text{Bl}_2 \mathbb{P}^2$



+ rel's

$\text{Bl}_3 \mathbb{P}^2$



+ rel's

3// QGIT

- Disclaimer Special case
- dim vector $(1, \dots, 1)$
 - α -generated stability
 - closed points

Defⁿ • A rep of Q, R is given by assigning $P_a \in \mathbb{F}$ to each arrow a such that rel's are satisfied

- A rep is stable if there is a nonzero path from α to each vertex i

- Rep(Q, R)^{stable} space of all stable reps

- A Morphism between two reps $(P_a) \xrightarrow{\theta} (\mu_a)$ is given by $\theta = (\theta_i \in \mathbb{F})$ i vertices, such that

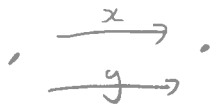
if $a: i \rightarrow j$

$$\begin{array}{ccc} & \xrightarrow{P_a} & \\ \theta_i \downarrow & & \downarrow \theta_j \\ & \xrightarrow{\mu_a} & \end{array}$$

- Isomorphism classes of stable reps are given by orbits of $\pi: \mathbb{F}^* \rightarrow G$

Defⁿ $M = \frac{\text{Rep}(Q, R)^{\text{stable}}}{G}$

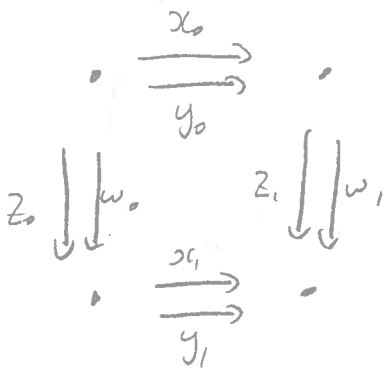
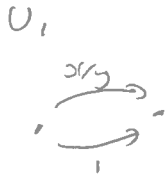
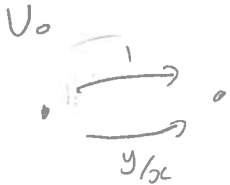
Examples



$$\text{Rep}(Q,R) = \mathbb{C}^2$$

$$\text{Rep}(Q,R)^{\text{stable}} = \mathbb{C}^2 \setminus \text{pt}$$

$$M = \text{Rep}(Q,R) / G = \mathbb{P}^1$$



$$\text{Rep}(Q,R) = \text{Spec } \mathbb{C}[x_i, y_i, w_i, z_i] / R$$

$$\begin{aligned} \text{Rep}(Q,R)^{\text{stable}} / G &= \frac{\mathbb{C}^2 \setminus \{0,0\}}{\mathbb{C}^\times} \times \frac{\mathbb{C}^2 \setminus \{0,0\}}{\mathbb{C}^\times} \dots \\ &\cong \mathbb{P}^1 \times \mathbb{P}^1 \end{aligned}$$

Summary • This works in these examples (King: Tilting bundles on some rational surfaces)

• Hopefully in others: Proj over affine, 1-dim fibres.